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ON INTUITIONISTIC FUZZY REGULAR α GENERALIZED CLOSED SETS

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ABSTRACT

The purpose of this paper is to introduce and study the concept of intuitionistic fuzzy regular α generalized closed set in intuitionistic fuzzy topological space. We investigate some of their properties.

KEYWORDS: Intuitionistic fuzzy set, Intuitionistic fuzzy topology, Intuitionistic fuzzy topological space, Intuitionistic fuzzy regular α generalized closed set.

INTRODUCTION

The theory of fuzzy sets was introduced by Zadeh [10] in 1965. Later, Chang [2] proposed fuzzy topology in 1967. After this, there have been several generalizations of notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets, introduced by Atanassov [1] is a generalization of fuzzy sets. In the last 25 years various concept of fuzzy mathematics have been extended for intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological spaces in 1997. This approach provided a wide field for investigation in the area of intuitionistic fuzzy topology.

In this paper, we introduce intuitionistic fuzzy regular α generalized closed set. We investigate some of their properties.

PRELIMINARIES

Definition 2.1:[1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ where the function $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2: [1] Let A and B be two IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

- $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$

$$d) A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$$

$$e) A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. The IFS $0 \sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1 \sim = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFS in X satisfying the following axioms:

- $0 \sim, 1 \sim \in \tau$,
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
- $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X . The complement A^c of an IFOS A in (X, τ) is called an intuitionistic fuzzy closed (IFCS in short) in X .

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by $\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \}$
 $\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}$

Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = (\text{int}(A))^c$ and $\text{int}(A^c) = (\text{cl}(A))^c$.

Definition 2.5: [5] An IFS A in an IFTS (X, τ) is said to be an

- intuitionistic fuzzy semi closed set (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$,
- intuitionistic fuzzy α closed set (IF α CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$,

- c) intuitionistic fuzzy pre closed set (IFPCS in short) if $cl(int(A)) \subseteq A$.

Definition 2.6: [9] An IFS A in an IFTS (X, τ) is said to be an

- a) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = cl(int(A))$,
 b) intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X ,
 c) intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X .

Definition 2.7: [7] An IFS A in an IFTS (X, τ) is intuitionistic fuzzy α generalized closed set (IF α GCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .

Definition 2.8: [8] Two IFSs A and B are said to be q -coincident ($A \text{ }_q \text{ } B$ in short) if and only if there exist an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.9: [8] Two IFSs A and B are said to be not q -coincident ($A \text{ }_q^c \text{ } B$ in short) if and only if $A \subseteq B^c$.

Definition 2.10:[4] An intuitionistic fuzzy point (IFP in short), written as $p_{(\alpha, \beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An IFP $p_{(\alpha, \beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

INTUITIONISTIC FUZZY REGULAR α GENERALIZED CLOSED SETS

In this section, we introduced intuitionistic fuzzy regular α generalized closed sets and studied some of their basic properties.

Definition 3.1: An IFS A of an IFTS (X, τ) is called intuitionistic fuzzy regular α generalized closed set (IFR α GCS in short) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X .

Example 3.2: Let $X = \{a, b\}$ and let $\tau = \{0\sim, U, G, 1\sim\}$ where $U = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$ where $\mu_a=0.4, \mu_b=0.2, \nu_a=0.6, \nu_b=0.7$ and $G = \langle x, (0.8, 0.8), (0.2, 0.2) \rangle$ where $\mu_a=0.8, \mu_b=0.8, \nu_a=0.2, \nu_b=0.2$. Let $A = \langle x, (0.2, 0.2), (0.8, 0.8) \rangle$ be any IFS in (X, τ) . Then $A \subseteq U$ where U is an IFROS in X . Now $\alpha cl(A) = \langle x, (0.2, 0.2), (0.8, 0.8) \rangle \subseteq U$. Therefore A is an IFR α GCS in (X, τ) .

Theorem 3.3: Every IFCS in (X, τ) is an IFR α GCS in (X, τ) but not conversely.

Proof: Let $A \subseteq U$ and U be an IFROS in X . We have $\alpha cl(A) = AUcl(int(cl(A))) = AUcl(int(A)) \subseteq AUcl(A) = AUA = A$, since by the hypothesis $cl(A) = A$. Therefore $\alpha cl(A) \subseteq U$. Hence A is an IFR α GCS in (X, τ) .

Example 3.4: Let $X = \{a, b\}$ and let $\tau = \{0\sim, U, G, 1\sim\}$ where $U = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle$ and $G = \langle x, (0.1, 0.2), (0.9, 0.8) \rangle$. Let $A = \langle x, (0.3, 0.1), (0.6, 0.7) \rangle$ be any IFS in (X, τ) . Then $A \subseteq U$ where U is an IFROS in X . Now $\alpha cl(A) = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle \subseteq U$. Therefore A is an IFR α GCS in X but not an IFCS in X , since $cl(A) = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle \neq A$.

Theorem 3.5: Every IFRCS in (X, τ) is an IFR α GCS in (X, τ) but not conversely.

Proof: Let $A \subseteq U$ and U be an IFROS in X . We have $\alpha cl(A) = AUcl(int(cl(A))) = AUcl(int(A))$, since every IFRCS is an IFCS $cl(A) = A$, and by hypothesis $A = cl(int(A))$. Therefore $\alpha cl(A) = AUA = A$, and hence $\alpha cl(A) \subseteq U$. Hence A is an IFR α GCS in (X, τ) .

Example 3.6: Let $X = \{a, b\}$ and let $\tau = \{0\sim, U, G, 1\sim\}$ where $U = \langle x, (0.6, 0.7), (0.3, 0.2) \rangle$ and $G = \langle x, (0.2, 0.2), (0.8, 0.7) \rangle$. Let $A = \langle x, (0.3, 0.1), (0.7, 0.7) \rangle$ be any IFS in (X, τ) . Then $A \subseteq U$ where U is an IFROS in X . Now $\alpha cl(A) = \langle x, (0.3, 0.2), (0.6, 0.7) \rangle \subseteq U$. Therefore A is an IFR α GCS in X but not an IFRCS in X , since $cl(int(A)) = 0\sim \neq A$.

Theorem 3.7: Every IF α CS in (X, τ) is an IFR α GCS in (X, τ) but not conversely.

Proof: Let $A \subseteq U$ and U be an IFROS in X . We have $\alpha cl(A) = AUcl(int(cl(A))) \subseteq AUA = A$, since by the hypothesis $cl(int(cl(A))) \subseteq A$. Therefore $\alpha cl(A) \subseteq U$. Hence A is an IFR α GCS in (X, τ) .

Example 3.8: Let $X = \{a, b\}$ and let $\tau = \{0\sim, U, G, 1\sim\}$ where $U = \langle x, (0.6, 0.7), (0.4, 0.2) \rangle$ and $G = \langle x, (0.1, 0.2), (0.6, 0.8) \rangle$. Let $A = \langle x, (0.1, 0.1), (0.6, 0.7) \rangle$ be any IFS in (X, τ) . Then $A \subseteq U$ where U is an IFROS in X . Now $\alpha cl(A) = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle \subseteq U$. Therefore A is an IFR α GCS in X but not an IF α CS in X , since $cl(int(cl(A))) = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle \not\subseteq A$.

Remark 3.9: Every IFR α GCSs and every IFPCSs are independent to each other.

Example 3.10: Let $X = \{a, b\}$ and let $\tau = \{0\sim, U, G, 1\sim\}$ where $U = \langle x, (0.5, 0.6), (0.4, 0.2) \rangle$ and $G = \langle x, (0.3, 0.2), (0.7, 0.8) \rangle$. Let $A = \langle x, (0.3, 0.2), (0.5, 0.6) \rangle$ be any IFS in (X, τ) . Then $A \subseteq U$ where U is an IFROS in X . Now $\alpha cl(A) = \langle x, (0.4, 0.2), (0.5, 0.6) \rangle \subseteq U$. Therefore A is an IFR α GCS in X , but not an IFPCS in X , since $cl(int(A)) = \langle x, (0.4, 0.2), (0.5, 0.6) \rangle \not\subseteq A$.

Example 3.11: Let $X = \{a, b\}$ and let $\tau = \{0\sim, U, G, 1\sim\}$ where $U = \langle x, (0.2, 0.3), (0.6, 0.7) \rangle$ and

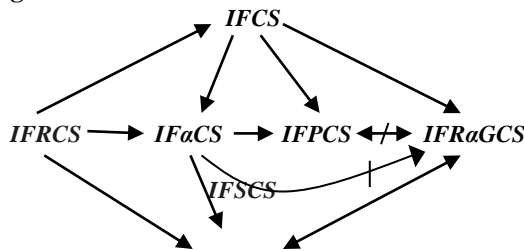
$G = \langle x, (0.8,0.7), (0.1,0) \rangle$. Let $A = \langle x, (0.1,0.2), (0.7,0.8) \rangle$ be any IFS in (X, τ) . Then $\text{cl}(\text{int}(A)) = 0 \sim \subseteq A$. Therefore A is an IFPCS in X but not an IFR α GCS in X , since $A \subseteq U$ where U is an IFROS in X but $\alpha\text{cl}(A) = \langle x, (0.6,0.7), (0.2,0.3) \rangle \not\subseteq U$.

Remark 3.12: Every IFR α GCSs and every IFSCSs are independent to each other.

Example 3.13: Let $X = \{a,b\}$ and let $\tau = \{0 \sim, U, G, 1 \sim\}$ where $U = \langle x, (0.6,0.7), (0.4,0.2) \rangle$ and $G = \langle x, (0.1,0.2), (0.7,0.8) \rangle$. Let $A = \langle x, (0.2,0.1), (0.6,0.7) \rangle$ be any IFS in (X, τ) . Then $A \subseteq U$ where U is an IFROS in X . Now $\alpha\text{cl}(A) = \langle x, (0.4,0.2), (0.6,0.7) \rangle \subseteq U$. Therefore A is an IFR α GCS in X but not an IFSCS in X , since $\text{int}(\text{cl}(A)) = \langle x, (0.1,0.2), (0.7,0.8) \rangle \not\subseteq A$.

Example 3.14: Let $X = \{a,b\}$ and let $\tau = \{0 \sim, U, G, 1 \sim\}$ where $U = \langle x, (0.5,0.2), (0.5,0.8) \rangle$ and $G = \langle x, (0.2,0.2), (0.8,0.8) \rangle$. Let $A = \langle x, (0.5,0.2), (0.5,0.8) \rangle$ be any IFS in (X, τ) . Then $\text{int}(\text{cl}(A)) = \langle x, (0.5,0.2), (0.5,0.8) \rangle = A$. Therefore A is an IFSCS in X but not an IFR α GCS in X , since $A = U$ where U is an IFROS in X , but $\alpha\text{cl}(A) = \langle x, (0.5,0.8), (0.5,0.2) \rangle \not\subseteq U$.

Figure:



In this diagram by “ $A \longrightarrow B$ ” we mean A implies B but not conversely and “ $A \longleftrightarrow B$ ” means A and B are independent of each other.

Theorem 3.15: If an IFS A is both IFSCS and IFCS in (X, τ) then A is an IFR α GCS in (X, τ) .

Proof: Let $A \subseteq U$ and U be an IFROS in X . We have $\alpha\text{cl}(A) = \text{AUcl}(\text{int}(\text{cl}(A))) \subseteq \text{AUcl}(A)$, since by the hypothesis $\text{int}(\text{cl}(A)) \subseteq A$. Also $\alpha\text{cl}(A) = \text{AUA}$, as $\text{cl}(A) = A$ by the hypothesis. Therefore $\alpha\text{cl}(A) = A \subseteq U$. Hence A is an IFR α GCS in (X, τ) .

Theorem 3.16: If an IFS A is both IFPCS and IFCS in (X, τ) then A is an IFR α GCS in (X, τ) .

Proof: Let $A \subseteq U$ and U be an IFROS in X . We have $\alpha\text{cl}(A) = \text{AUcl}(\text{int}(\text{cl}(A))) = \text{AUcl}(\text{int}(A))$, since by the hypothesis $\text{cl}(A) = A$. Also $\alpha\text{cl}(A) \subseteq \text{AUA}$, as $\text{cl}(\text{int}(A)) \subseteq A$ by the hypothesis, $\alpha\text{cl}(A) = A \subseteq U$. Hence A is an IFR α GCS in (X, τ) .

Theorem 3.17: If an IFS A is both IFROS and IFCS in (X, τ) then A is an IFR α GCS in (X, τ) .

Proof: Let $A \subseteq U$ and U be an IFROS in X . We have $\alpha\text{cl}(A) = \text{AUcl}(\text{int}(\text{cl}(A))) = \text{AUcl}(A)$, since by the

hypothesis $\text{int}(\text{cl}(A)) = A$. Also $\alpha\text{cl}(A) = \text{AUA}$, as $\text{cl}(A) = A$ by the hypothesis, $\alpha\text{cl}(A) = A \subseteq U$. Hence A is an IFR α GCS in (X, τ) .

Theorem 3.18: If an IFS A is both IFSCS and IFGCS in (X, τ) then A is an IFR α GCS in (X, τ) .

Proof: Let $A \subseteq U$ and U be an IFROS in X . Since every IFROS is an IFOS, U is an IFOS. We have $\alpha\text{cl}(A) = \text{AUcl}(\text{int}(\text{cl}(A))) \subseteq \text{AUcl}(A)$, since by the hypothesis $\text{int}(\text{cl}(A)) \subseteq A$. Also $\alpha\text{cl}(A) \subseteq \text{cl}(A)$. By the hypothesis $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X . Therefore $\alpha\text{cl}(A) \subseteq U$. Hence A is an IFR α GCS in (X, τ) .

Remark 3.19: The union of two IFR α GCS in an IFTS (X, τ) need not be IFR α GCS in general.

Example 3.20: Let $X = \{a,b\}$ and let $\tau = \{0 \sim, U, G, H, I, J, K, 1 \sim\}$ is an IFT on (X, τ) . Where $U = \langle x, (0.6,0.7), (0.3,0.2) \rangle$, $G = \langle x, (0.4,0.2), (0.4,0.8) \rangle$, $H = \langle x, (0.2,0.2), (0.4,0.8) \rangle$, $I = \langle x, (0.4,0.2), (0.4,0.7) \rangle$, $J = \langle x, (0.4,0.2), (0.6,0.8) \rangle$ and $K = \langle x, (0.2,0.2), (0.6,0.8) \rangle$. Let $A = \langle x, (0.6,0.7), (0.4,0.2) \rangle$ and $B = \langle x, (0.4,0.7), (0.3,0.2) \rangle$ be any two IFS in (X, τ) . Then $A \subseteq U$ where U is an IFROS in X . $\alpha\text{cl}(A) = \langle x, (0.6,0.7), (0.4,0.2) \rangle \subseteq U$ and then $B \subseteq U$ where U is an IFROS in X . $\alpha\text{cl}(B) = \langle x, (0.4,0.7), (0.3,0.2) \rangle \subseteq U$. Therefore A and B are IFR α GCS in X but $A \cup B = \langle x, (0.6,0.7), (0.3,0.2) \rangle$ is not an IFR α GCS in X , since $A \cup B \subseteq U$ but $\alpha\text{cl}(A \cup B) = \langle x, (0.6,0.8), (0.2,0.2) \rangle \not\subseteq U$.

Theorem 3.21: Let A be an IFR α GCS in an IFTS (X, τ) and $A \subseteq B \subseteq \alpha\text{cl}(A)$ then B is an IFR α GCS in (X, τ) .

Proof: Let $B \subseteq U$ and U be an IFROS in X . Then $A \subseteq U$, since $A \subseteq B$. As A is an IFR α GCS in X , $\alpha\text{cl}(A) \subseteq U$. But by the hypothesis, $B \subseteq \alpha\text{cl}(A) \Rightarrow \alpha\text{cl}(B) \subseteq \alpha\text{cl}(A) \subseteq U$. This implies $\alpha\text{cl}(B) \subseteq U$. Hence B is an IFR α GCS in (X, τ) .

Theorem 3.22: Let (X, τ) be an intuitionistic fuzzy topological space and A be an IF α CS of (X, τ) . Then A is an IFR α GCS in (X, τ) if and only if $A \text{ }_q \text{ }^\circ \text{ } F \Rightarrow \alpha\text{cl}(A) \text{ }_q \text{ }^\circ \text{ } F$ for every IFRCFS F of (X, τ) .

Necessity: Let F be an IFRCFS of X , and $A \text{ }_q \text{ }^\circ \text{ } F$. Then $A \subseteq F^\circ$, by definition and F° is IFROS in X . Therefore $\alpha\text{cl}(A) \subseteq F^\circ$ because A is an IFR α GCS in X . Hence $\alpha\text{cl}(A) \text{ }_q \text{ }^\circ \text{ } F$.

Sufficiency: Let U be an IFROS of X such that $A \subseteq U$. Then $A \text{ }_q \text{ }^\circ \text{ } (U^\circ)$ and U° is an IFRCFS in X . Hence by hypothesis, $\alpha\text{cl}(A) \text{ }_q \text{ }^\circ \text{ } (U^\circ)$. Therefore $\alpha\text{cl}(A) \subseteq (U^\circ)^\circ = U$. Hence A is an IFR α GCS in (X, τ) .

Theorem 3.23: If A is both an IFROS and IFR α GCS in (X, τ) . Then A is an IFRGCS in (X, τ) .

Proof: Let $A \subseteq U$ and U be an IFROS in X . We have $\text{cl}(A) = \text{AUcl}(A)$, By hypothesis $\text{int}(\text{cl}(A)) = A$ as

A is an IFROS. This implies $\text{cl}(A) = \text{AUcl}(\text{int}(\text{cl}(A))) = \alpha\text{cl}(A)$, By the hypothesis $\alpha\text{cl}(A) \subseteq U$. This implies $\text{cl}(A) \subseteq U$. Hence A is an IFRGCS in (X, τ) .

Theorem 3.24: If A is both an IFPOS and IFR α GCS in (X, τ) . Then A is an IFRGCS in (X, τ) .

Proof: Let $A \subseteq U$ and U be an IFROS in X. We have $\text{cl}(A) = \text{AUcl}(A)$, By hypothesis $A \subseteq \text{int}(\text{cl}(A))$ as A is an IFPOS. This implies $\text{cl}(A) \subseteq \text{AUcl}(\text{int}(\text{cl}(A))) = \alpha\text{cl}(A)$, By the hypothesis $\alpha\text{cl}(A) \subseteq U$. This implies $\text{cl}(A) \subseteq U$. Hence A is an IFRGCS in (X, τ) .

Theorem 3.25: If A is an IFROS and an IFR α GCS in (X, τ) . Then A is an IF α CS in (X, τ) .

Proof: As $A \subseteq A$, by the hypothesis, $\alpha\text{cl}(A) \subseteq A$. But we have $A \subseteq \alpha\text{cl}(A)$. This implies $\alpha\text{cl}(A) = A$. Hence A is an IF α CS in (X, τ) .

Theorem 3.26: Let (X, τ) be an IFTS. Then every IFS in (X, τ) is an IFR α GCS if and only if IF α O(X) = IF α C(X).

Necessity: Suppose that every IFS in (X, τ) is an IFR α GCS. Let $U \in \text{IFRO}(X)$, then $U \in \text{IF}\alpha\text{O}(X)$ and by the hypothesis, $\alpha\text{cl}(U) \subseteq U \subseteq \alpha\text{cl}(U)$. This implies $\alpha\text{cl}(U) = U$. Therefore $U \in \text{IF}\alpha\text{C}(X)$. Hence $\text{IF}\alpha\text{O}(X) \subseteq \text{IF}\alpha\text{C}(X)$. Let $A \in \text{IF}\alpha\text{C}(X)$, then $A^c \in \text{IF}\alpha\text{O}(X) \subseteq \text{IF}\alpha\text{C}(X)$. That is, $A^c \in \text{IF}\alpha\text{C}(X)$. Therefore $A \in \text{IF}\alpha\text{O}(X)$. Hence $\text{IF}\alpha\text{C}(X) \subseteq \text{IF}\alpha\text{O}(X)$. Thus $\text{IF}\alpha\text{O}(X) = \text{IF}\alpha\text{C}(X)$.

Sufficiency: Suppose that $\text{IF}\alpha\text{O}(X) = \text{IF}\alpha\text{C}(X)$. Let $A \subseteq U$ and U be an IFROS. Then $U \in \text{IF}\alpha\text{O}(X)$ and $\alpha\text{cl}(A) \subseteq \alpha\text{cl}(U) = U$, since $U \in \text{IF}\alpha\text{C}(X)$, by hypothesis. Therefore A is an IFR α GCS in (X, τ) .

Theorem 3.27: Let A be an IFR α GCS in (X, τ) and $p(\alpha, \beta)$ be an IFP in X such that $\text{int}(p(\alpha, \beta)) \supseteq \alpha\text{cl}(A)$, then $\text{cl}(\text{int}(p(\alpha, \beta))) \supseteq A$.

Proof: Let A be an IFR α GCS and let $\text{int}(p(\alpha, \beta)) \supseteq \alpha\text{cl}(A)$. If $\text{cl}(\text{int}(p(\alpha, \beta))) \supseteq A$ then by Definition 2.9, $A \subseteq [\text{cl}(\text{int}(p(\alpha, \beta)))]^c$ where $[\text{cl}(\text{int}(p(\alpha, \beta)))]^c$ is an IFROS. Then by hypothesis, $\alpha\text{cl}(A) \subseteq [\text{cl}(\text{int}(p(\alpha, \beta)))]^c = \text{int}(\text{cl}(p(\alpha, \beta)))^c \subseteq \text{cl}(p(\alpha, \beta))^c = (\text{int}(p(\alpha, \beta)))^c$. This implies $\alpha\text{cl}(A) \supseteq \text{int}(p(\alpha, \beta))$. Therefore by Definition 2.9, $\text{int}(p(\alpha, \beta)) \supseteq \alpha\text{cl}(A)$, which is a contradiction to the hypothesis. Hence $\text{cl}(\text{int}(p(\alpha, \beta))) \supseteq A$.

Theorem 3.28: Let $F \subseteq A \subseteq X$ where A is an IFROS and an IFR α GCS in (X, τ) . Then F is an IFR α GCS in A if and only if F is an IFR α GCS in (X, τ) .

Necessity: Let U be an IFROS in X and $F \subseteq U$. Also let F be an IFR α GCS in A. Then clearly $F \subseteq A \cap U$ and $A \cap U$ is an IFROS in A. Hence the α closure of F in A, $\alpha\text{cl}_A(F) \subseteq A \cap U$. By Theorem 3.25, A is an IF α CS. Therefore $\alpha\text{cl}(A) = A$ and the α closure of F in X, $\alpha\text{cl}(F) \subseteq \alpha\text{cl}(F) \cap \alpha\text{cl}(A) = \alpha\text{cl}(F) \cap A = \alpha\text{cl}_A(F) \subseteq A \cap U \subseteq U$. That is $\alpha\text{cl}(F) \subseteq U$ whenever $F \subseteq U$. Hence F is an IFR α GCS in A.

Sufficiency: Let V be an IFROS in A such that $F \subseteq V$. Since A is an IFROS in X, V is an IFROS in X. Therefore $\alpha\text{cl}(F) \subseteq V$, since F is an IFR α GCS in X. Thus $\alpha\text{cl}_A(F) = \alpha\text{cl}(F) \cap A \subseteq V \cap A \subseteq V$. Hence F is an IFR α GCS in A.

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